PROF. PAULO
AULA 26
ARCOS

| X | Sen(x) | cos(x) | tg(x) |
|---|---|---|---|
| $30^0$ | $\frac{1}{2}$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{3}}{3}$ |
| $45^0$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{2}}{2}$ | 1 |
| $60^0$ | $\frac{\sqrt{3}}{2}$ | $\frac{1}{2}$ | $\sqrt{3}$ |

<u>ADIÇÃO E SUBTRAÇÃO DE ARCOS</u>

sen(a + b) = sen(a).cos(b) + cos(a).sen(b)
sen(a – b) = sen(a).cos(b) - cos(a).sen(b)

cos(a + b) = cos(a).cos(b) - sen(a).sen(b)
cos(a - b) = cos(a).cos(b) + sen(a).sen(b)

tg(a + b) = $\frac{tg(a)+tg(b)}{1-tg(a).tg(b)}$

tg(a - b) = $\frac{tg(a)-tg(b)}{1+tg(a).tg(b)}$

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Exemplo$_1$:
Calcule sen($75^0$)
Resolução:
sen($75^o$) = sen($45^o$ + $30^o$) =
= sen($45^o$).cos($30^o$) + cos( $45^o$).sen($30^o$) =

$$= \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2}+\frac{\sqrt{2}}{2}.\frac{1}{2} =$$

$$\frac{\sqrt{6}}{4}+\frac{\sqrt{2}}{4} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Exemplo$_2$:

Calcule $\cos(105^o)$

Resolução:

$\cos(105^o) = \cos(45^o + 60^o) =$

$= \cos(45^o).\cos(60^o) - \text{sen}(45^o).\text{sen}(60^o) =$

$$= \frac{\sqrt{2}}{2}.\frac{1}{2} - \frac{\sqrt{2}}{2}.\frac{\sqrt{3}}{2} =$$

$$\frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4} = \frac{\sqrt{2}-\sqrt{6}}{4}$$

Exemplo$_3$:

Calcule o valor de $\cos(90^0 + x)$

Resolução:

$\cos(90^0 + x) = \cos(90^0).\cos(x) - \text{sen}(90^0).\text{sen}(x) =$

$= 0.\cos(x) - 1.\text{sen}(x) =$

$= -\text{sen}(x)$

Exemplo$_4$:

Sabendo-se que tg(a) = 3 e tg(b) = 2,

calcule tg(a + b)

Resolução:

$$tg(a+b) = \frac{tg(a)+tg(b)}{1-tg(a).tg(b)} =$$

$$= \frac{3+2}{1-3.2} =$$

$$= \frac{5}{1-6} =$$

$$= \frac{5}{-5} =$$

$= -1$

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<u>ARCO DUPLO</u>

Definições:

$\text{sen}(2x) = \text{sen}(x + x) = \text{sen}x.\cos x + \cos x.\text{sen}x = 2.\text{sen}x.\cos x$

$\cos(2x) = \cos(x + x) = \cos x.\cos x - \text{sen}x.\text{sen}x = \cos^2 x - \text{sen}^2 x$

Lembrando que $\text{sen}^2 x + \cos^2 x = 1$

1) $\cos^2 x = 1 - \text{sen}^2 x$

$\cos(2x) = \cos^2 x - \sin^2 x = 1 - \sin^2 x - \sin^2 x = 1 - 2.\sin^2 x$

2) $\sin^2 x = 1 - \cos^2 x$

$\cos(2x) = \cos^2 x - \sin^2 x = \cos^2 x - (1 - \cos^2 x) = \cos^2 x - 1 + \cos^2 x =$
$= 2.\cos^2 x - 1$

$\text{tg}(2x) = \text{tg}(x + x) = \dfrac{tgx + tgx}{1 - tgx.tgx} = \dfrac{2.tgx}{1 - tg^2 x}$

Resumo das fórmulas

$\text{sen}(2x) = 2.\text{sen}(x).\cos(x)$

$\cos(2x) = \begin{cases} \cos^2(x) - \text{sen}^2(x) \\ 2.\cos^2(x) - 1 \\ 1 - 2.\text{sen}^2(x) \end{cases}$

$\text{tg}(2x) = \dfrac{2tg(x)}{1 - tg^2(x)}$

Exemplo$_1$:

Sabendo-se que $\cos(x) = \dfrac{1}{3}$ e que $0 \le x \le 2\pi$, calcule $\cos(2x)$

Resolução:

$\cos(2x) = 2.\cos^2(x) - 1 =$

$= 2.\left(\dfrac{1}{3}\right)^2 - 1$

$= 2.\dfrac{1}{9} - 1 =$

$= \dfrac{2}{9} - 1 =$

$= \dfrac{2 - 9}{9} =$

$= \dfrac{-7}{9}$

Exemplo$_2$:

Simplifique a expressão:

$\cos^4(x) - \sin^4(x)$

Resolução:

$\cos^4(x) - \text{sen}^4(x) =$

$= [\cos^2(x) - \text{sen}^2(x)].[\cos^2(x) + \text{sen}^2(x)] =$

$= [\cos^2(x) - \text{sen}^2(x)].1 =$

$= \cos(2x).1 =$

$= \cos(2x)$

Exercícios:

1) Calcule o valor de $\text{sen}(90^0 + x).\cos(180^0 - x)$

2) Calcule o valor de $tg15^0$

3) O valor de $(\sec 20^0.\text{sen}\,20^0 + \text{cossec}\,20^0.\cos 20^0).\text{sen}\,40^0$ é:

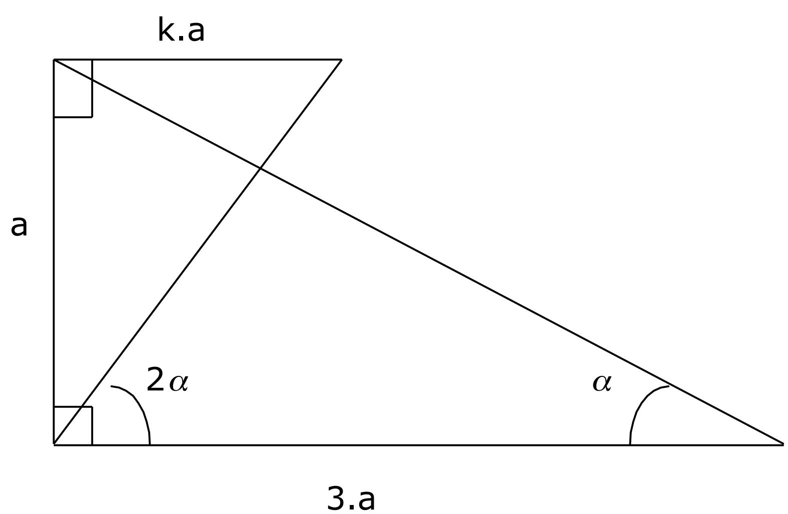
a) $\text{sen}40^0$ b) $\cos 40^0$ c) 2 d) 1 e) 0

4) (UFMG) – A expressão $\frac{2.tgx}{1+tg^2x}$ é idêntica a:

a) tg(2x) b) cos(2x) c) sen(2x) d) 2.sen(x) e) sen(x).cos(x)

5) (UFMA) – De acordo com os dados da figura abaixo, o valor de k é:

a) 3/2 b) 1/3 c) 4/3 d) _ e) 1/4




RESOLUÇÃO:

1) Calcule o valor de $\text{sen}(90^0 + x).\cos(180^0 - x)$

Resolução:

$\text{sen}(90^0 + x).\cos(180^0 - x) =$

$= (\text{sen}90^0.\cos x + \cos 90^0.\text{sen}x).(\cos 180^0.\cos x + \text{sen}180^0.\text{sen}x) =$

$= (1.\cos x + 0.\text{sen}x).(-1.\cos x + 0.\text{sen}x) =$

$= (\cos x).(-\cos x) = -\cos^2 x$

2) Calcule o valor de $tg15^0$

Resolução:

tg($15^0$) = tg($60^0$ - $45^0$) = $\frac{tg60^0 - tg45^0}{1 + tg60^0.tg45^0} = \frac{\sqrt{3} - 1}{1 + \sqrt{3}.1}$ =

$$\frac{\sqrt{3}-1}{\sqrt{3}+1}.\frac{\sqrt{3}-1}{\sqrt{3}-1} = \frac{(\sqrt{3})^2 - 2.\sqrt{3} + 1^2}{(\sqrt{3})^2 - 1^2} = \frac{3 - 2\sqrt{3} + 1}{3 - 1} = \frac{4 - 2\sqrt{3}}{2} = 2 - \sqrt{3}$$

3) O valor de $(\sec 20^0.\text{sen}\, 20^0 + \text{cos}\sec 20^0.\cos 20^0).\text{sen}\, 40^0$ é:

a) sen$40^0$ b) cos$40^0$ c) 2 d) 1 e) 0

Resolução:

$$(\sec 20^0.\text{sen}\, 20^0 + \text{cos}\sec 20^0.\cos 20^0).\text{sen}\, 40^0 =$$

$$(\frac{1}{\cos 20^0}.\text{sen}\, 20^0 + \frac{1}{\text{sen}\, 20^0}.\cos 20^0).\text{sen}(2.20^0) =$$

$$(\frac{\text{sen}\, 20^0}{\cos 20^0} + \frac{\cos 20^0}{\text{sen}\, 20^0}).2.\text{sen}\, 20^0.\cos 20^0 =$$

$$(\frac{\text{sen}^2\, 20^0 + \cos^2 20^0}{\cos 20^0.\text{sen}\, 20^0}).2.\text{sen}\, 20^0.\cos 20^0 =$$

$$(\frac{1}{\cos 20^0.\text{sen}\, 20^0}).2.\text{sen}\, 20^0.\cos 20^0 =$$

$$(1).2 = 2$$

Resposta c

4) (UFMG) – A expressão $\frac{2.tgx}{1 + tg^2 x}$ é idêntica a:

Resolução:

a) tg(2x) b) cos(2x) c) sen(2x) d) 2.sen(x) e) sen(x).cos(x)

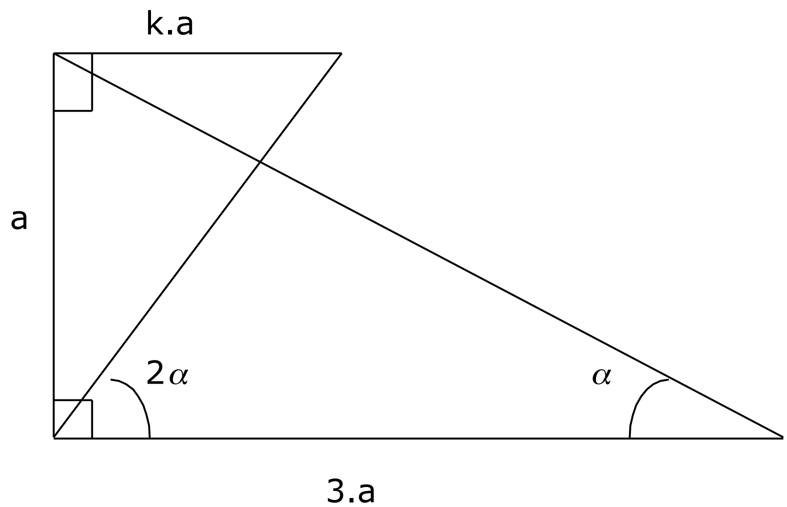
$$\frac{2.tgx}{1 + tg^2 x} = \frac{2.\frac{\text{sen}\, x}{\cos x}}{1 + \frac{\text{sen}^2 x}{\cos^2 x}} = \frac{\frac{2.\text{sen}\, x}{\cos x}}{\frac{\cos^2 x + \text{sen}^2 x}{\cos^2 x}} = \frac{\frac{2.\text{sen}\, x}{\cos x}}{\frac{1}{\cos^2 x}} = \frac{2.\text{sen}\, x}{\cos x}.\frac{\cos^2 x}{1} =$$

$$= 2.\text{sen}\, x.\cos x = \text{sen}(2x)$$

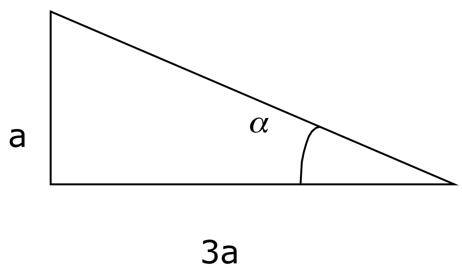
Resposta c

5) (UFMA) – De acordo com os dados da figura abaixo, o valor de k é:

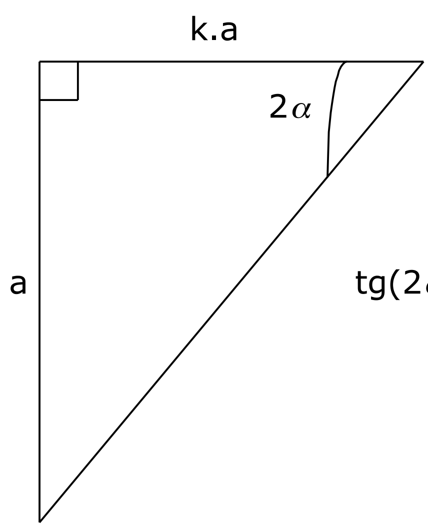
a) 3/2 b) 1/3 c) 4/3 d) _ e) 1/4




Resolução:




$$tg(\alpha) = \frac{cat.oposto}{cat.adjacente} = \frac{a}{3a} = \frac{1}{3}$$




$$tg(2\alpha) = \frac{cat.oposto}{cat.adjacente} = \frac{a}{k.a} = \frac{1}{k}$$

$$tg(2\alpha) = \frac{2.tg\alpha}{1 - tg^2\alpha}$$

$$\frac{1}{k} = \frac{2.\frac{1}{3}}{1 - (\frac{1}{3})^2}$$

$$\frac{1}{k} = \frac{\frac{2}{3}}{1 - \frac{1}{9}}$$

$$\frac{1}{k} = \frac{\frac{2}{3}}{\frac{8}{9}}$$

$$\frac{1}{k} = \frac{2}{3}.\frac{9}{8}$$

$$\frac{1}{k} = \frac{3}{4}$$

k = $\frac{4}{3}$

Resposta c